Two kinds of paradoxes

- Paradoxes of Motion (PM)
- Paradoxes of Plurality (PP)

But in the sources it is attested: [Simplicius, Phys., 139 (5)]: “In his book, in which many arguments are put forward, he shows in each that stating a plurality comes down to stating a contradiction.”

Zeno’s Paradox of Plurality

PP (Simplic., Phys., 140 (34)): “So if it is a plurality, it by necessity will be many small ones and many large ones ... so many small(s) as to have no magnitude, so many larger(s) as to be unbounded (apart?)”

Zeno’s ‘Division through and through’

i) it goes on “infinitely”;
ii) it is symmetrical (proportion 1:2);
iii) it results somehow in parts with and without magnitude;
iv) it is a simultaneous procedure;
v) it pops up in all his paradoxes.

→ Axiom of Choice (well-ordering)

A well-ordered continuum

The large Zeno-semilattice \( \langle \leq \rangle \) is the ideal completion of \( \langle \mathbb{Z}, < \rangle \), the small Zeno-lattice. Every node of the \( \omega \)-divisional level (or every possible branch in the \( \mathbb{N}^\mathbb{N} \) generation), i.e., every element \( x \) generated by \( \exists \) and \( \leq \) a unique sequence \( f(x) = (x_n)_{n \in \mathbb{N}} \). The finite ideals \( x \) can be ordered by inclusion. They all have a supremum. The supremum of the infinite \( \omega \)-chain of \( \leq \) a they are a part of the maximal element \( 1 \) \( 3 \) \( \lambda \) \( \leq \) \( \omega \). By virtue of \( \sigma \), each \( \omega \)-chain defines a unique order on \( \mathbb{Z} \). Every ordinality on \( \mathbb{Z} \) coincides with a chain and no other chains do appear. This gives the orderstructure of \( \mathbb{P} \) (IN). The Zenoian semilattice \( \langle \leq \rangle \) is a directed and complete partial order or apos.

But \( \mathbb{Z} \) is as well a total order \( \leq \) by the canonical numbering of the parts, all elements will be comparable: \( \forall x, y \in x < y \vee y < x \). This total order is lexicographic. A well-order \( \leq \) is a total order in which every non-empty subset of \( \mathbb{Z} \) has a least element. This is the case here: all ideals \( \leq \) included in each unique sequence \( f(x) = (x_n)_{n \in \mathbb{N}} \) are finite and non-empty.

→ construction of a well-ordered continuum

References

Abraham, W., The nature of Zeno’s Argument Against Plurality in [DK 298 1], in Phronesis, 47, 1972.
Sierpiński, W., Hypothése du Continu, Z Subwencji Funduszu Kultury Narodowej, Warsawa/Lwów, 1934.
Zeno's Paradoxes. “Refutes” Zeno by walking; not sufficient to just reject the conclusion (we must show what's wrong with the argument, show a premise is false). Dichotomy Argument. 1. All segments (finite lengths) can be divided into two segments C1. All segments can be divided into segments without limit C1.5. All segments are composed of an infinity of segments 2. All segments have finite length 3. The length of any segment = sum of lengths of segments that compose it C2. The length of any segment = an infinite sum of finite lengths 4. All finite sums of finite quantities are infinite C3. All segments are... Â The cardinal number of the set of natural numbers is the smaller infinite cardinal. Non-Denumerable. The cardinality of the real numbers. Problems: 1. Tortoises of the genus Gopherus have been clocked walking at speeds of about 0.1 m/s. This means that a tortoise should be able to complete a 100m race in about 1000s (∼16.67 minutes). In order to get to the 100m mark, however, the tortoise must complete infinitely many tasks: she must reach the 50m mark, then reach the 75m mark, then reach the 87.5m mark, and so forth. (For each i ≥ 1, she must reach the 100(1 − 21i) mark.) How is this possible? How can a tortoise complete infinitely many tasks (one for each i ≥ 1) in a finite amount of time (1000s)? 2. Lazy wants to run from A to...